Kant's Argument from the Applicability of Geometry

Waldemar Rohloff, University of Missouri, St Louis

Abstract

In this paper I develop a reading of Kant's argument from geometry based on distinguishing the roles of pure versus applied geometry. Once these roles are properly distinguished, I argue that the argument from geometry is not susceptible to the problems concerning the development and applications of non-Euclidean geometry, which are often thought to undermine the argument.

Introduction

In the Transcendental Exposition of the Concept of Space, early in the *Critique of Pure Reason*, Kant aims to establish that space is transcendentally ideal by presenting what has come to be known as 'the argument from geometry'. The centrality and importance of this claim about space makes this argument a cornerstone of Kant's exposition and defense of transcendental idealism. Despite its significance, however, the argument from geometry has a rather poor reputation, for it is often assumed that it rests on Kant's belief that Euclidean geometry is *a priori* and necessarily true of objects in space. In the

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study that follows, I will develop a reading of Kant's argument that shows this assumption to be incorrect.

Traditionally, the argument from geometry has been thought to run as follows: Kant observes that pure geometry is a necessarily true, synthetic *a priori* body of knowledge, and contends that the only explanation of our geometrical knowledge is that space is a pure form of sensible intuition, and therefore, transcendentally ideal.[1] In the first section of the paper, I develop an alternative interpretation on which the argument from geometry turns on the application of geometry to empirical objects, and not our knowledge of *pure* geometry.

In the second section of the paper, my interpretation is used to show that the argument from geometry is not dependent on the necessity of Euclidean geometry. Secondly, by appealing to Kant's notion of an undetermined object, I demonstrate that the argument from geometry does not require applied geometry to be *a priori*. Of course, I accept that Kant believes Euclidean geometry is necessary and *a priori*. In this respect, the aims of the second section of the paper are reconstructive rather than interpretive. To borrow a phrase from Paul Guyer, I provide an account of ‘Kant's literal accomplishment’ rather than ‘Kant's own intention’ (Guyer 1987, 442). My aim is to show that the argument Kant provides does not depend essentially on Kant’s conception of geometry as necessary and *a priori*. In this respect, my reconstruction provides a defense of Kant’s argument from geometry, and thereby, his transcendental idealism about space. The paper shows that in the course of arguing for the transcendental idealism of space, Kant made assumptions about geometry that are stronger than his argument requires.

1 See (Broad 1978); (Strawson 1966); (Russell 1937); (Guyer 1987), though there are important differences between these interpretations.

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1. Transcendental Idealism and the Transcendental Exposition of Space

In the Transcendental Aesthetic, Kant first formulates the doctrine that space is transcendentally ideal when he asks:

> Now what are space and time? Are they actual entities? Are they only determinations or relations of things, yet ones that would pertain to them even if they were not intuited, or are they relations that attach only to the form of intuition alone, and thus to the subjective constitution of our mind, without which these predicates could not be ascribed to anything at all? (A23/B37)[2]

Kant's doctrine of the transcendental ideality of space consists of two claims, one positive, one negative. The positive claim is that space is a 'form of sensible intuition'. The negative claim is that spatial predicates cannot be ascribed to things thought of independently from the constitution of our minds (i.e. things in themselves); in other words, space is *nothing but* a form of sensible intuition.

It is a notorious challenge to adjudicate the relationship between these two claims. Some commentators interpret Kant as deriving the positive claim from the negative (Guyer 1987). Others interpret him as deriving the negative claim from the positive. As an example of the latter, Henry Allison argues that it is impossible to define either an identity, correspondence or similarity relation between space as a form of sensibility and space as a property of things in themselves (Allison 2004, 128-132). As a result, there is no meaningful sense in which the same space can be both a form of sensibility and a property of

[2] All of my citations of Kant are in reference to the *Akademie Edition* (Kant 1902), though I make use of the translation by Guyer and Wood (Kant 1998), with occasional modifications.

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things in themselves. In this paper, I will rest the argument for the negative claim on Allison's analysis.[3] As a result, my main concern will be to show how Kant's argument from geometry establishes that space is a form of sensibility. The reliance on Allison's argument will be assessed in more detail in the concluding section of the paper.

The argument from geometry is one of several arguments Kant offers to establish the transcendental ideality of space.[4] I take the argument from geometry to occur primarily in the second and third paragraphs of the Transcendental Exposition of the Concept of Space.[5] Because of their importance, I quote these two paragraphs here in full:

> Geometry is a science that determines the properties of space synthetically, and yet *a priori*. What, then, must the representation of space be for such a cognition of it to be possible? It must originally be intuition; for from a mere concept no propositions can be drawn that go beyond the concept, which, however, happens in geometry (Introduction, V). But this intuition must be encountered in us *a priori*, i.e. prior to all perception of an object, thus it must be pure, not empirical intuition. For geometrical

[3] (B56) contains Kant's straightforward formulation of this position.

[4] I make no claim that the argument from geometry is Kant's primary or only argument for transcendental idealism. Henry Allison constructs an argument based on the Metaphysical Exposition, which largely bypasses the Transcendental Exposition and the argument from geometry (Allison 2004).

[5] Some writers attribute the argument from geometry to other passages in the Aesthetic, for instance, B66 is sometimes cited (van Cleve 1999, 34-35); (Guyer 1987, 365). I resist seeing this passage as part of the argument from geometry. One reason for this is that the argument's location in the text makes it an odd candidate for Kant's canonical argument for transcendental idealism. The argument at B66 occurs after the Metaphysical and Transcendental Expositions and the Conclusions section, in which Kant explicitly draws and argues for the thesis of transcendental idealism. The conclusion of the argument at B66 is the negative component of transcendental idealism: space does not apply to things in themselves. This makes the argument at B66 important for Guyer's interpretation of Kant's transcendental idealism, on which Kant first argues for the negative claim, with the positive claim following from the negative.

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> propositions are all apodictic, i.e. combined with consciousness of their necessity; e.g., space has only three dimensions; but such propositions cannot be empirical or judgments of experience, nor inferred from them (Introduction, II).
>
> Now how can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of the latter can be determined *a priori*? Obviously not otherwise than insofar as the intuition has its seat merely in the subject, as its formal constitution for being affected by objects, and thereby acquiring *immediate representation*, i.e., *intuition*, of them, thus only as the form of outer *sense* in general. (B41)

Although the two paragraphs together constitute the argument from geometry, each contains a distinctive part of the argument. The focus of the second paragraph is given by the question Kant poses at the start of the passage: 'Geometry is a science which determines the properties of space synthetically, and yet *a priori*. What, then, must the representation of space be, for such a cognition of it to be possible?' (B40). Importantly, Kant does not wish to draw out consequences about the nature of space itself. Rather, at this point, Kant is asking about *our representation* of space. He wants to know what that representation must be like in order for geometry to be possible.[6] His answer is that our representation of space is an *a*

[6] It might be objected that in the second paragraph of the Transcendental Exposition Kant is discussing space itself, not merely our representation of it. There Kant begins with the assertion that 'Geometry is a science which determines the properties of space synthetically, and yet *a priori*'. Furthermore, he wishes to know what our representation of space must be like in order for geometry to be possible. But geometry apparently constitutes knowledge of space itself. In that case it may seem that more is under discussion in that paragraph than merely our representation of space. In a sense the above objection has it right. Since the distinction between our representation of space and space itself does not characterize distinct ontological entities, there is a sense in which space itself is already under discussion in paragraph two of the Transcendental Exposition. But to emphasize that space itself is under discussion here because of

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priori intuition. This is the same claim as that of the Metaphysical Exposition, and as such, it falls short of the assertion that space is transcendentally ideal.[7] Nevertheless, we will see that it sets an important foundation for the rest of the argument from geometry.

In the third paragraph of the Transcendental Exposition, Kant begins with a different question. He asks: ‘How can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of these objects can be determined *a priori*?’ (B41). Notice that in the first clause Kant refers to the conclusion of the previous paragraph. When Kant asks: ‘How can an outer intuition inhabit the mind that precedes the objects themselves?’ the explicit reference to ‘outer intuit-tion’ and ‘mind’ *(Gemüt)* suggests that Kant is referring to our representation of space. It is in the second clause of the above

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the Kant's identification of space itself with our representation of it, is to bring Kant's conclusions to bear earlier parts of his argument. At this stage, he has not yet identified space with our representation of it, and so the focus on our representation of space is justified by the argumentative structure of the text.

[7] I take the Metaphysical and Transcendental Expositions to be offering different arguments for the same basic claim, namely that our representation of space is an *a priori* intuition (though the Transcendental Exposition goes beyond this claim by also making claims about space itself). This reading differs from (Shabel 2004). In that paper, Shabel argues that Kant assumes the results of the Metaphysical Exposition, and instead tries to show how geometry ‘flows from’ our *a priori* intuition of space. On Shabel’s reading, the Transcendental Exposition provides a ‘bridge’ between the results of the Metaphysical Exposition and the full-blown transcendental idealism of the later sections of the Transcendental Aesthetic. I resist reading Kant’s argument in this way for the following reasons. First, the question framing the second paragraph of the Transcendental Exposition: ‘What then must the representation of space be for such a cognition of it to be possible?’ indicates Kant is trying to establish something about our representation of space, rather than assuming something about it. Secondly, Kant begins the second paragraph with the assertion that ‘Geometry is a science that determines the properties of space synthetically and yet *a priori*’. This fact is then used to argue that our representation of space must be an *a priori* intuition. For instance, Kant points out that ‘from a mere concept, no propositions can be drawn that go beyond the concept, which however, happens in geometry’. Kant then concludes by explaining that our representation of space ‘must originally be intuition’. Here Kant is reasoning from the nature of geometry to a conclusion about our representation of space. Such a pattern of argument is characteristic of a regressive argument, rather than the progressive argument Shabel attributes to Kant.

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question - 'in which the concept of these objects can be determined *a priori*' - where Kant finally moves to considerations that bear on the nature of space itself.[8] The essence of Kant's question is: how is it that we can have *a priori* knowledge of an object based upon our *a priori*, intuitive representation of space? Kant's answer to this question is that space is transcendentally ideal. He tells us that the above is possible only if 'intuition has its seat merely in the subject, as its formal constitution for being affected by objects, and thereby acquiring *immediate representation*, i.e., *intuition*, of them, thus only as the form of outer *sense* in general' (B41). In other words, space is subjective and nothing but a form of sensibility.

The division canvassed above, between the second and third paragraphs of the Transcendental Exposition, partitions the argument from geometry into two separate arguments: an argument from pure geometry and an argument from applied geometry. This division might raise questions among readers of Kant, for it is sometimes asserted, even as a commonplace, that Kant does not have a distinction between pure and applied geometry.[9] Hence, I will first argue that Kant indeed has such a distinction. Then I will show that this distinction tracks the division between the second and third paragraphs of the

[8] Charles Parsons suggests that the above argument is not so much a part of the argument from geometry as it is a part of the Metaphysical Exposition (Parsons 1992, 81-82). He takes Kant to be giving an argument from the possibility of *a priori* intuition (rather than *a priori* knowledge) in the above passage. He understands Kant to be suggesting that the only explanation for our *a priori* intuition of space is that space is transcendentally ideal. The problem with this reading is that it does not square well with Kant's second requirement on a transcendental exposition: that of showing 'that these cognitions are only possible under the presupposition of a given way of explaining this concept'. In accord with this, Kant's concern in the above passage is not to explain the existence of an *a priori* intuition of space, but rather with the way that this *a priori* intuition affords us knowledge of objects. The question posited in the first sentence of the above passage is not merely: how can there be an *a priori* intuition of space? Rather, the question is: how can there be an *a priori* intuition which provides us with *a priori* knowledge?

[9]See for instance the introductory remarks of Carnap in (Reichenbach 1958). There it is alleged that it is precisely for lack of such a distinction that Kant's philosophy of geometry falters.

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Transcendental Exposition. Finally, I will argue that the argument from pure geometry turns on the necessity and certainty of pure geometry, whereas the argument from applied geometry is focused on the *a priori* nature of applied geometry.

Those who claim Kant lacks a distinction between pure and applied geometry may have in mind the modern distinction between an uninterpreted formal system expressed in quantifier logic (pure geometry) and an interpretation of that formal system (applied geometry). Michael Friedman's work provides an explanation of why Kant lacks the above distinction. He points out that without the aid of polyadic quantification, Kant cannot formulate a formal system adequate for the needs of geometrical proof (Friedman 1992, 56-66). According to Friedman, given Kant's impoverished logic, pure intuition is required to represent certain features of space such as its order properties. This makes the very notion of a formal system of geometrical proof impossible for Kant. But without this, it follows that Kant has no distinction between an interpreted and uninterpreted formal system, and thus, no possibility of making the modern distinction between pure and applied geometry.

However, just because Kant cannot make the *modern* distinction between pure and applied geometry, it does not follow that he has no such distinction at all. To the contrary, there is substantial evidence pointing to Kant's recognition of a distinction between pure and applied geometry. In the early modern presentations of geometry familiar to Kant, it is common to distinguish applied mathematics, which deals with topics such as mechanics, optics, astronomy and architecture, from pure mathematics, which deals with pure magnitudes.[10] Furthermore, several of Kant's pre-critical philosophical and scientific works attest to his concern with the use of geometry in the description of nature, as this issue connects with the controversy over the compatibility of Leibnizean metaphysics

[10] See (Wolff 1965) as discussed in (Shabel 2003, 41-43).

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with natural science.[11] In *Thoughts on the True Estimation of Living Forces,* Kant suggests a fundamental division between mathematics and nature; mathematics is associated with the quantitative study of motion, while metaphysics is associated with its qualitative aspects (AA 1:139-142). In Kant's *Physical Monadology*, a distinction between pure geometry and its application to the empirical world arises again. In that work, Kant sets out to reconcile the apparently contradictory conceptions of geometrical and metaphysical space. Finally, moving into the critical period, Kant distinguishes in a number of places between pure mathematics and its application to the empirical world (B15, A40/B57, A157/B196, A165/B206, B147, AA 4:283-4, AA 4:287-8, AA 4:471, AA 18:241).

In the critical period, Kant no longer distinguishes mathematics and nature as in *Living Forces*, nor mathematical from metaphysical space as in *Physical Monadology*. Rather, in the critical period, Kant's distinction between pure and applied geometry rests on the contrast between our representation of space and space itself. Kant's critical turn establishes that space itself is constituted by our representation. Because of this identification, it is crucial to emphasize that the distinction between our representation of space and space itself does not characterize two ontologically distinct entities. Instead the distinction refers to two different ways of considering the same thing: space. To use Fregean terminology, 'our representation of space' and 'space itself' have the same reference, but a different sense. To consider space from the perspective of our representation, is to focus on the way we subjectively represent space, while ignoring what makes our representation objective. Kant adduces such features in the Metaphysical Exposition (B39), for example, 'space is *represented as* an infinite given magnitude' (italics added). By contrast, to consider space from the perspective of space itself is to focus on the features that make it objective. We see this, for example, when Kant writes

[11] See (Schönfeld 2000) for an extensive discussion.

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that space 'is a necessary condition of all the relations within which objects can be intuited as outside us' (A27/B43).

Kant's distinction between pure and applied geometry rests on these two different ways of considering space. Just as the distinction between our representation of space and space itself is not an ontological distinction, so too, Kant's distinction between pure and applied geometry is not ontological. Rather, the distinction between pure and applied geometry characterizes two distinct ways of considering the same thing: geometry. Fregean terminology applies again. 'Pure geometry' and 'applied geometry' have the same reference, but a different sense. This difference in sense comes out in Kant's remarks on what pure geometry would be, if it were not possible to apply it to the empirical world. Pure geometry, without the possibility of application, is characterized as 'a mere game' (A239/B298) or 'figment of the brain' (A157/B198). It is only because applied geometry is possible that our mathematical concepts have any objectivity (A239/B298, A157/B198, B147, AA 4:287). The contrast between the (potentially) object-less character of pure geometry and the robust objectivity of applied geometry illuminates Kant's critical-period distinction between pure and applied geometry. It can be thought of as the difference between geometry as practiced in the pure representation of space of the geometer, irrespective of any relation to empirical intuition, as opposed to the application of geometry to the world of empirical experience.[12]

What we have are two different conceptions of space and two different conceptions of geometry. Each conception of

[12] It must be emphasized that when Kant speaks of pure geometry he does not mean something purely subjective as opposed to applied geometry which is objective. Kant believes that pure geometry is perfectly objective. But explaining how this objectivity is possible requires the argument of the *Critique of Pure Reason* and is not an assumption of his thinking about geometry. Friedman (who sometimes emphasizes the lack of a distinction between pure and applied geometry) in fact himself makes this observation (Friedman 1992, 94). That the preceding matches the structure of Kant's thought on the matter is made especially clear in the *Prolegomena* (AA, 4:287).

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space is aligned with a conception of geometry. On the one hand, we have our representation of space aligned with (non-applied) pure geometry. Both are considered in subjective terms, ignoring their capacity for objectivity: we have space 'as it is represented' and geometry as a 'figment of the brain'. On the other hand, we have space itself aligned with applied geometry. Here space is considered in terms of its capacity for objectivity, as is geometry.

Given that Kant in fact has a distinction between pure and applied geometry, and that this distinction is aligned with the distinction between our representation of space and space itself, it follows that we can divide Kant's argument in the Transcendental Exposition as proposed. The second paragraph, which argues that our representation of space is an *a priori* intuition, contains an argument from pure geometry. The third paragraph, which argues that space itself is transcendentally ideal, contains an argument from applied geometry.

One further observation remains to be made. Once we divide Kant's argument from geometry as proposed, we see that in the argument from pure geometry, Kant emphasizes the necessity of pure geometry, whereas in the argument from applied geometry, Kant emphasizes the *a priori* knowledge gained through applied geometry. Though '*a priori*' and 'necessary' are coextensive for Kant, they differ in sense. As we will see, this difference in sense means they function differently in the argument from geometry.

Beginning with the argument from pure geometry, we see that considerations of necessity and certainty play a central role in Kant's characterization of geometrical knowledge. In the second paragraph of the Transcendental Exposition, Kant says: 'For geometrical propositions are one and all apodictic, i.e. combined with the consciousness of their necessity' (B41). From this he infers that our intuitive representation of space 'must be encountered in us *a priori*' (B41). The same argument occurs at A24, the part of the A edition Metaphysical

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Exposition that grew into the Transcendental Exposition of the B edition. There Kant writes:

> If this representation of space were a concept acquired *a posteriori*, which was drawn out of general outer experience, the first principles of mathematical demonstration would be nothing but perceptions. They would therefore have all the contingency of perception, and it would not even be necessary that only one straight line lie between two points, but experience would merely always teach that.

All the elements present in the second paragraph of the Transcendental Exposition are present here as well. Kant draws the same conclusion: our representation of space is *a priori*. As evidence, he argues that if our representation were not *a priori*, then geometrical necessity would be impossible. Since geometrical propositions are necessary, it follows that our representation of space must be *a priori.*

The stress on necessity, rather than the *a priori,* plays an important polemical role at this point in the argument. For Kant, the necessity of geometry has an immediate connection with the content of geometrical propositions. This is evident in Kant's formulation: 'geometrical propositions are one and all apodictic, i.e. combined with the consciousness of their necessity' (B41). For Kant, necessity is an inseparable component of geometrical propositions. If we grasp a geometrical proposition, we are immediately 'conscious' that it is necessary. This differs from our awareness that geometrical propositions are *a priori*, which is inferred. Kant sometimes even entertains the possibility that geometrical judgments could be *a posteriori* (A24, B15, A48/B56). He always rules out this possibility, but he does so by appeal to the necessity or certainty of geometry. He repeatedly argues that if geometry were *a posteriori,* then the necessity or certainty of geometry would be lost (*ibid*). For

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Kant, the *a priori* character of geometry is inferred from the necessity of geometry, while the necessity of geometry is unassailable and immediately given with the geometrical proposition itself. So Kant's appeal to necessity in the argument from pure geometry reflects our firmer grasp on the necessity of geometry.[13]

The argument from applied geometry is located in the third paragraph of the Transcendental Exposition. There we see a change, with Kant emphasizing *a priori* geometrical knowledge, rather than the necessity of geometry.[14] Kant asks: 'how can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of these objects can be determined *a priori*'?[15] The mention of objects turns our attention towards applied geometry, for it is applied geometry that deals with objects, strictly speaking.[16] It is clear then, that Kant's concern in the third paragraph is the question of how *a priori* knowledge of applied geometry is possible. For Kant, the only viable explanation is the thesis that space is transcendentally ideal. The explanation he gives is: '[outer intuition] has its seat merely in the subject, as its formal constitution for being affected by objects and thereby acquiring immediate representation, i.e. intuition, of them, thus only as the form of outer sense in general' (B41). We can see the positive component of

[13] Moreover, if Kant were to focus solely on the *a priori* character of pure geometry, he would be showing that our *a priori* knowledge of geometry requires an *a priori* intuition of space. This is not an empty conclusion, but insisting that *a priori* representation is required for *a priori* knowledge is not terribly surprising. It does more to bolster our impression of the power of Kant's notion of *a priori* intuition, if he can show that the necessity and certainty of pure geometry also requires an *a priori* intuition of space.

[14] See also (AA 4:282).

[15] Though the third paragraph of the Transcendental Exposition does not contain explicit mention of geometry, the context of the passage makes it evident that geometry remains under discussion. In particular, what a transcendental exposition is supposed to accomplish – provide an explanation of *a priori* cognitions – makes it clear that geometry must still be under consideration.

[16] Here I am assuming the view that for Kant, abstract mathematical objects are not genuine objects. This perspective has been formulated in (Friedman 1992, 94, Parsons 1983, 147-149, Parsons 1992, 136-140).

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the transcendental ideality thesis, namely that space is a form of sensibility, when space is characterized as the 'formal constitution for being affected by objects' and as 'the form of outer sense in general'. We can see the negative component, namely that space is nothing other than a form of sensibility, when Kant says 'it has its seat *merely* in the subject' and '*only* as the form of outer sense' [my emphasis].[17] Much the same argument can be found in the Conclusions section following the Transcendental Exposition. There Kant writes:

> Space is nothing other than merely the form of all appearances of outer sense, i.e., the subjective condition of sensibility, under which alone outer intuition is possible for us. Now since the receptivity of the subject to be affected by objects necessarily precedes all intuitions of these objects, it can be understood how the form of all appearances can be given in the mind prior to all actual perceptions, thus *a priori*, and how as pure intuition, in which all objects must be determined, it can contain principles of their relations priori to all experience. (A26/B42)

Here we see all of the same elements as in the third paragraph of the Transcendental Exposition. Kant explicitly mentions objects *(Gegenstände)* indicating that it is applied

[17] Shabel writes: 'It is only upon concluding the "Transcendental Exposition" that Kant explicitly introduces transcendental idealism, claiming that space represents no property of things in themselves and equivalently, that space is nothing more than a subjective representation' (Shabel 2004, 207). Shabel does allow that Kant argues for the positive component of transcendental idealism in the Transcendental Exposition (Shabel 2004, 207). Her reticence may lie with the absence of an 'explicit' argument for the negative component of the transcendental idealism thesis. I agree there is a lack of an explicit argument here, and so my reading does not dispute Shabel on this point. However, I should add, I do take Kant to be doing more than merely stating the transcendental idealism thesis, as he does, for instance in the opening paragraph of the Metaphysical Exposition. In the Transcendental Exposition, the thesis is not merely proposed or described, but rather is presented as an integral part of an argument and explanation of geometrical knowledge.

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geometry that is under discussion. Kant's concern is to explain the *a priori* character of our knowledge, not its certainty or necessity. The claim that space is a form of sensibility is taken to explain how such *a priori* knowledge is possible. Finally, we also have the negative component of the transcendental idealism thesis: 'space is nothing other than' the form of sensibility.

Kant supports his idealism claim by comparing his account of space with that of the 'mathematical investigators of nature', by which he means to refer to a Newtonian view, and the 'metaphysicians of nature', by which he means to refer to the views of Leibniz and Wolff (B56-7).[18] Kant claims that neither of these positions can provide a satisfactory account of the applicability of geometry. According to Kant, the mathematical investigators of nature must posit space and time as 'two eternal and infinite self-subsisting non-entities' (B56). Kant concedes that this posit does allow for the applicability of geometry to nature, but only at the cost of violating the boundaries of the applicability of geometry. The 'metaphysicians of nature' claim that space and time are 'abstracted from experience though confusedly represented in this abstraction' (B57). According to Kant, this has as a consequence that they cannot account for the *a priori* nature of mathematical cognitions, and further, they cannot 'bring the propositions of experience into necessary accord with [mathematics]' (B57).

More important than Kant's objections to the views of others is the positive explanation he gives of applied geometry. The question Kant raises at the opening of the third paragraph of the Transcendental Exposition – 'how can an outer intuition inhabit the mind that precedes the objects themselves, and in which the concept of these objects can be determined *a priori*' – ties together the results of the second paragraph, with the explanation offered in the third. In the second paragraph, Kant establishes that our representation of space is an *a priori* intuition. In the background is Kant's belief that pure geometry 'flows from'

[18] For further discussion of these arguments see (Shabel 2005).

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our *a priori* intuition of space.[19] With these pieces in place, the argument from geometry is intended to answer the following: *how can a discipline based solely on our a priori representation of space (pure geometry) give us a priori knowledge through its application to actual objects?* The importance of this question is that it turns our attention to the distinctively *a priori* status of pure geometry, and seeks an explanation for how such a distinctively *a priori* discipline applies to the world of experience.[20] Kant's answer is that space must have a special role to play in cognition, as the form in which all outer objects are given to us. This is a powerful answer to the question posed, one which will be discussed further in the following section.

Here I briefly summarize the results of this section. It was argued that Kant has a distinction between pure and applied geometry, and we can use this distinction to mark out two distinct parts of the argument from geometry. Each of these arguments is directed towards different conclusions. The argument from pure geometry aims to establish that our representation of space is an *a priori* intuition, and it does so through an emphasis on the necessity of pure geometry. The argument from applied geometry aims to establish that space is a form of sensibility, and does so through an emphasis on *a priori* knowledge. Precedent for a division along these lines comes from Kemp-Smith, who writes:

> Then from the apodictic character of geometry [Kant] infers that space exists in us as pure and *a priori*; no experience can ever reveal necessity. But geometry also

[19] This is a big assumption. In my view, this is an assumption Kant allows himself in the Transcendental Exposition. The argument underlying this assumption is filled out most extensively in the Transcendental Doctrine of Method section of the *Critique of Pure Reason*, where Kant gives his most detailed account of construction in pure intuition. An alternative account of the Transcendental Exposition is found in (Shabel, Kant's Argument from Geometry 2004). Shabel argues that in the Transcendental Exposition, Kant does not assume, but rather aims to establish, that geometry flows from our *a priori* intuition of space.

[20] See also (AA 18:271).

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> exists as an applied science; and to account for our power of anticipating experience, we must view space as existing only in the perceiving subject as the form of its sensibility. (Kemp-Smith 1923, 111)[21]

Kemp-Smith's apt description of the argument from applied geometry as turning on our 'power of anticipating experience' nicely characterizes Kant's chief *explanadum* in the argument from applied geometry.

2. Resolving Some Problems with Euclidean Geometry

In this section I develop two significant consequences of the reading of Kant's argument from geometry set out in the previous section. First, I show that Kant's argument does not depend essentially on the necessity of Euclidean geometry, thus defending the argument from the objection that it suffers due to this alleged dependence. Then I show that the argument from geometry can be read as independent from a commitment to geometry as an *a priori* true theory of space.

Kant's argument from geometry is frequently taken to depend on the necessary truth of Euclidean geometry as a theory of space, and is dismissed precisely for this presumption (Guyer 1987, 365, Strawson 1966, 70). Were Kant's argument to depend on such a commitment, the development of non-Euclidean geometries and their application in modern physics would provide ample reason for such dismissal. However, the analysis of the argument from geometry offered in this paper shows that Kant's argument need not depend on the necessity of geometry. As has been discussed, Kant's argument from geometry diverges into two distinct arguments, one that emphasizes the necessity of pure geometry, and one that emphasizes the *a priori* nature of applied geometry. Impor-

[21] A similar account of Kant's argument is found in (Vaihinger 1976, 268-270).

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tantly, the step where Kant argues that space is a form of sensibility concerns only the latter. In that argument, Kant seeks to explain how a discipline based upon our *a priori* representation of space (pure geometry) gives us knowledge in its application to the empirical world (applied geometry). Kant's commitment to the necessity of geometry only comes into play in the argument from pure geometry, where Kant argues that our representation of space is an *a priori* intuition. We do need this result for the argument from applied geometry to work, but it is available elsewhere, in particular from arguments in the Metaphysical Exposition.[22] Thus, the conclusion that Kant's argument from geometry depends on the necessity of geometry needs to be carefully qualified. When Kant uses the argument from geometry to argue that space is a form of sensibility, he does not rely on the necessity of geometry, but only on its *a priori* status.

Of course, Kant does believe that everything *a priori* is necessary and vice versa; for him, the notions are coextensive. This means that Kant's commitment to the *a priori* status of geometry is also a commitment to its necessity. However, this need not undermine the point made above. Recall from the previous section that for Kant, '*a priori*' and 'necessary' differ in sense. When '*a priori*' appears in the argument from applied geometry, it is the sense of the term that is doing argumentative work. In that argument Kant wishes to explain 'our power of anticipating experience', to borrow Kemp-Smith's phrase, rather than the immediately given necessity of geometric propositions. In this respect, the necessity of geometry plays no critical role in the argument from geometry. So, for the purposes of the reconstruction in this paper, Kant's commitment to the necessity of geometry can be set aside.

Still, setting aside Kant's commitment to the necessity of geometry does not remove the weight of the evidence provided

[22] See (Allison 2004, Warren 1998) for how the Metaphysical Exposition establishes that our representation of space is an *a priori* intuition.

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by modern physics against his argument. What modern physics suggests is that geometry, in so far as it is a theory about actual space, is not *a priori* either. Kant's position can be reconstructed in a number of ways that offer solutions to this problem. One proposal is to allow that geometry does not describe actual space, but rather 'the phenomenal field'; geometry characterizes how things appear to us visually, not how they are physically (Strawson 1966).[23] Alternatively, we could try to weaken the notion of the *a priori*, as in the relative *a priori*, recently expounded by Michael Friedman (Friedman 2001).

Here I would like to suggest an alternative approach, one where Kant's argument can be run independently of a commitment to applied geometry as a body of *a priori* truths about space. As we have seen, Kant's argument from geometry turns on the question:

> (1) How can a discipline based upon our *a priori* representation of space (pure geometry) give us *a priori* knowledge through its application to actual objects?

I propose we change Kant's question by eliminating the second mention of the *a priori*. The question we pose instead is:

> (2) How can a discipline based upon our *a priori* representation of space (pure geometry) give us knowledge through its application to actual objects?

[23] A problem raised with this view concerns whether pure intuition can distinguish between cases of minute geometrical variation (Friedman 1992); (Kitcher 1992); (Parsons 1983).

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I will argue that Kant's answer to the first question – space is the form of sensibility – remains a compelling answer to the second.

To see this, we must first appreciate why (2) is a significant question. It might be remarked that (2) lacks the intrigue of (1), for if applied geometry is *a posteriori*, then it can be explained in the same terms as other empirical knowledge. However, this remark fails to appreciate that geometry, like other mathematical disciplines, has a distinctive methodology of proof and abstract *a priori* reasoning, which contrasts with the methods of experiment and observation in empirical sciences. Given this contrast, we are left to ask why the *a priori* methods of mathematics extend so fortuitously into empirical disciplines.[24] This is the question (2) raises.

Kant's transcendental idealism suggests the following answer to (2): in order to account for the applicability of geometry, our representation of space must have a special role in cognition. In particular, our representation of space serves as a form of sensibility, in which all outer objects are given to us. In this way, we explain why *a priori* reasoning with our representation of space (pure geometry) transfers over to the behavior of empirical objects in space. As the development of non-Euclidean geometry shows, our *a priori* representation yields a richer variety of geometrical spaces than Kant realized. For this reason, not all *a priori* reasoning transfers over to the empirical world, and we need empirical intervention to find out which geometry is correct. But the question remains why *any a priori* reasoning about space should transfer over to the empirical world. Kant's transcendental idealism provides the answer. And so, Kant's explanation of applied geometry remains compelling, even if applied geometry is *a posteriori*.

[24] This question is related to Wigner's famous 'miracle' of applied mathematics (Wigner 1960).

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To see the power of this explanation of applied geometry, we can compare it with an alternative.[25] Take for instance a formalist-inspired perspective, such as that of Hempel, who describes the application of mathematical theories as follows:

> For the purposes of applying any one of these non-arithmetical disciplines to some specific field... of empirical science, it is therefore necessary first to assign to the primitives some specific meaning and then to ascertain whether in this interpretation the postulates turn into true statements. (Hempel 1984, 392)

For Hempel, the application of mathematical theories is the result of providing a true empirical interpretation of content-less formal theories. As an explanation of the applicability of geometry, this falls short, for it provides no account of why patterns of *a priori* spatial reasoning transfer over to the empirical world. Empirical interpretation is perhaps what happens in applications, but is not an account of why those applications work.

The above argument shows that it is not essential that applied geometry be *a priori* for the argument from geometry to go through. Clearly this differs from Kant's own view, for Kant believes applied geometry to be *a priori*. However, I propose that the above reconstruction of the argument from geometry can be fitted within the structure of the *Critique of Pure Reason*

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[25] The claim in this reconstruction is that Kant's transcendental idealism provides the best explanation of the applicability of geometry. Fully substantiating this claim requires comparing Kant's explanation to all other competing explanations, a task that goes beyond the scope of this paper. What the reader should take away from the discussion in this section is that Kant's explanation is a live option, and at least *prima facie,* a strong one. This is sufficient to support the defense of transcendental idealism attempted in this paper, which aims to show that Kant's argument from geometry is not a non-starter simply because geometry is contingent *a posteriori.* In other words, if the argument from geometry ultimately fails because there is an alternative, better explanation of the applicability of geometry, this raises a problem with the argument that goes beyond the problem this paper was designed to address.

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without upsetting key tenets of Kant's transcendental idealism about space. In this sense it is a viable account of what Kant accomplished, if not of what he intended.[26]

This proposal depends on highlighting the explanatory role of the Transcendental Aesthetic. To see what this is, we can turn to Kant's notion of an appearance, as presented in the Transcendental Aesthetic. There Kant defines an appearance as: 'the undetermined object of an empirical intuition' (A20/B34). The important point to notice here is Kant's characterization of an appearance as an 'undetermined object' *(unbestimmte Gegenstand)*.[27] Determination of an object is achieved in Kant's system through the conceptual activity of the understanding.[28] When Kant refers to appearances as 'undetermined', it is because the role of the Transcendental Aesthetic is to provide an analysis of the contribution of sensibility to cognition. This analysis is achieved by abstracting away the contribution of the understanding, thus leaving us with an undetermined object. As Kant tells us:

> In the transcendental aesthetic we will therefore first isolate sensibility by separating off everything that the

[26] In the *Critique of Pure Reason*, Kant takes a strong position about the necessary and *a priori* character of applied geometry. However, early in his career he allows that it is an empirical matter as to which of a variety of possible geometries applies to space. In his *True Estimation of Living Forces*, Kant argues that the three dimensional character of space is a consequence of the inverse square law of attraction (AA, I: 24-25).

[27] See (Sutherland 2005) for a discussion of the contrast between the determinate and indeterminate in cognition, and the role this plays in Kant's philosophy of mathematics.

[28] Here I mean to remain neutral as to whether the forms of intuition of the Aesthetic are independent of the understanding, or whether, as (Longuenesse 1998) has argued, they are themselves the result of a pre-categorial synthesis. What I am committed to is that determination happens through the categories, and that the faculty of sensibility described in the Aesthetic is thought of as determinable with respect to the categories. Kant tells us: 'synthesis is still an exercise of spontaneity, which is determining, and not like sense, merely determinable, and can thus determine the form of sense *a priori* in accordance with the unity of apperception, the imagination is to this extent a faculty for determining the sensibility' (B151-2).

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> understanding thinks through its concepts, so that nothing but empirical intuition remains. Second, we will then detach from the latter everything that belongs to sensation, so that nothing remains except pure intuition and the mere form of appearances, which is the only thing that sensibility can make available *a priori*. (A22/B36)

An important consequence of abstracting away the contribution of the understanding is that we are left with something less than genuine cognition; all cognition requires the activity of the understanding.[29] Thus, what Kant is left to explain in the Aesthetic is an aspect of cognition that can be considered prior to the conceptual activity of the understanding. It is controversial what intuition alone contributes to cognition. Questions arise as to whether it is intuition or the understanding that contributes order, multiplicity and measure, among other features of space.[30] I cannot address these issues here. My contention is only that whatever intuition contributes need not determine a choice between Euclidean and non-Euclidean geometries.

Given its location in the Aesthetic, the argument from geometry should be understood as based on this abstraction from the conceptual contributions of the understanding. Though the argument begins with genuine cognitions of space, we quickly abstract away the work of the understanding. Abstracting away from the conceptual contribution of the understanding means that we no longer have genuine geometrical cognitions, for as Kant details in the Axioms of Intuition, geometrical cognition requires a 'synthesis of the productive imagination'

[29] At (A426/B454) Kant does allow that we can intuit an 'indeterminate quantum as a whole'. However, intuiting an indeterminate magnitude (quantum) would still not count as intuiting an indeterminate object. For Kant, magnitudes of any kind require the representation of plurality, which corresponds to one of the categories of the understanding. Thus, Kant's concession that we can intuit an indeterminate magnitude does not mean he allows that we can cognize an indeterminate object. See (Sutherland 2004) for discussion of magnitudes.

[30] See (Falkenstein 1995) for discussion.

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(A163/B204). Now the reconstructed argument from geometry can be put in fully Kantian terms: if space is a form of sensibility, then some geometry will be applicable to determinate objects found in that space, once those objects are synthesized by some discursive intellect. How exactly the understanding goes about synthesizing those objects is irrelevant in the Aesthetic. However the understanding does it, some geometry will apply. In this sense, we have shown that Kant's belief that applied geometry is synthetic *a priori* does not play an essential role in Kant's argument from geometry. Hence, we have Kant's argument from the applicability of geometry.

Importantly, I am not claiming that the applicability of geometry is fully explained by Kant in the Transcendental Aesthetic. In the Transcendental Analytic, Kant tells us that the synthetic activity of the understanding is what 'makes pure mathematics in its complete precision applicable to objects of experience' (A165/B206). Kant believed that Euclidean geometry was the completely precise description of space. On the reading given here, we can avoid this commitment. The claim is not that Kant was right about Euclidean geometry. Rather, it is that only a more general, and at the same time less 'precise', aspect of the applicability of geometry is explained in the Aesthetic.[31] The explanation that we are given - that space is a form of sensibility - assures us that some geometry will apply to objects in space, after the understanding has performed its synthetic activity. What is accounted for is manifested in the emphasis on applicability in the title of this paper. The title is intended to underscore that on the strongest reading of Kant's argument, what is accounted for need not be construed as a body of theoretical propositions, but rather as a capacity shared

[31] The attempt to find more general and at the same time weaker criteria for Kant's philosophy of geometry to satisfy is not new. See for instance (Russell 1996); (Brittan 1978).

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by many geometries but realized by only one, namely the capacity to apply to the world of experience.[32]

3. Conclusions

In the first section of this paper, I claimed that the argument from geometry, especially as it occurs in the Transcendental Exposition, bifurcates into two distinct parts. These two distinct parts are directed towards different though complementary conclusions. The first part emphasizes the necessity and certainty of pure geometry, and seeks to establish that our representation of space is an *a priori* intuition. The second part emphasizes the *a priori* nature of applied geometry, and seeks to establish that space itself is transcendentally ideal.

In the second section of this paper, I showed how this way of understanding Kant's argument from geometry allows us to avoid the objection that his argument falters based on a commitment to the necessity of Euclidean geometry. As we have seen, Kant's argument only emphasizes the necessity of geometry when he argues that our representation of space is an *a priori* intuition. When Kant argues that space is a form of sensibility his focus turns to our *a priori* knowledge of applied geometry. Next, I argued that, since the Transcendental Aesthetic only deals with undetermined objects and not full cognitions, Kant's task in the Transcendental Aesthetic is to explain the capacity for some geometry to apply to experience, rather than to explain the fully determinate propositions of applied geometry. With this perspective in hand, we saw how to read Kant's argument from geometry in a way that does not conflict with the *a posteriori* status of applied geometry.

[32] Friedman denies that non-Euclidean geometry is even potentially applicable, from Kant's point of view (Friedman 1992). This claim rests on Friedman's understanding of Kant's notions of real and logical possibility. Emily Carson offers a criticism of Friedman's interpretation and an analysis of Kant's notions of real and logical possibility, on which non-Euclidean geometry is logically possible for Kant (Carson 1997). I rest my claims here on Carson's analysis.

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As a defense of Kant's transcendental idealism, my argument is subject to a significant caveat. This caveat concerns my reliance on Allison's argument that if space is a form of sensibility then space cannot be a property of things in themselves. This assumption fits into my reconstruction of Kant's argument in the following way. In the reconstruction, we conclude that space is a form of sensibility because that is the best explanation for the applicability of geometry. The step where we infer that space does not apply to things in themselves bypasses considerations directly related to geometry, and comes secondarily, from the claim that space is a form of sensibility. To fully substantiate Kant's claim that space does not apply to things in themselves is a daunting task, for it requires the elimination of the famous neglected alternative. This task goes beyond the scope of this paper.

However, if the reader does not grant Allison's argument, the reading of Kant's argument from geometry developed here still offers valuable insight. In the first place, even if we deny Allison's inference, the Kantian argument for the claim that space is a form of sensibility remains intact. This argument makes a point that is of interest from the perspective of the philosophy of geometry generally, whether or not transcendental idealism is true. For it suggests that in order to explain the applicability of geometry we must grant our representation of space a distinguished role, as the *a priori* form in which outer objects are given to us. This shows us that any formalist conception of geometry will be incomplete, as long as the concept of a geometrical space is treated as arbitrary. Secondly, the reading developed here carries out the defense of transcendental idealism promised at the beginning of the paper: namely, it shows that Kant's argument for transcendental idealism does not fall short due to his belief that geometry is necessary and *a priori*, as is commonly thought.[33] Rather, if it

[33] If there is some other way in which the argument from geometry falls short, this goes beyond the scope of the defense in this paper. See note 25.

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falls short, it falls short at the inference from space as a form of sensibility to space not being a property of things in themselves.[34]

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